Algebra Readiness through Deeper Learning in Middle School

HOW TEACHERS CAN EMPOWER STUDENTS TO ACHIEVE WITH CONFIDENCE

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Introduction

Given the right conditions for learning, every student can successfully understand algebra. The ideal conditions involve students thinking critically, working collaboratively, communicating their ideas effectively, and directing their own learning as they understand and master core academic content. In addition, when students develop a growth mindset—an empowered disposition that enables them to persevere through difficulties and setbacks—they can work through challenges in algebra to meet curricular goals.

The combination of these conditions is referred to as Deeper Learning. By using these strategies to engage students in algebraic thinking as early as kindergarten, we can have a tremendous impact on each student’s long-term success and achievement in mathematics.

When students are engaged in Deeper Learning, algebraic reasoning can be nurtured and promoted at any age. Using the principles of Deeper Learning both in the classroom and with digital technology, educators can help elementary, middle, and high school students overcome barriers to learning algebra. The possibilities and opportunities provided by blended learning models using appropriate digital tools and adaptive capabilities can support differentiation, acceleration, and remediation to enable student understanding. For teachers with a large number of students, adaptive learning technologies extend the capacity to individualize lessons and enhance student learning.

This white paper discusses the six competencies of Deeper Learning and shares strategies for using these competencies to make learning algebra more empowering, effective, and meaningful.
Barriers to algebra learning

Generally considered the gateway that opens up the opportunity for high school graduation, college, and careers, an Algebra 1 course formalizes many concepts and skills that are essential for understanding and using the language of mathematics. The ability to reason algebraically helps students think logically, identify patterns, form conclusions, construct arguments, and solve new and unfamiliar problems. Without being able to use these critical abilities in both school and life, students lack the necessary foundation from which to build the capacity to understand and respond to the challenges they face in their lives, communities, and world. Given the importance of these skills, it’s essential to define the barriers to learning algebra that Deeper Learning can help overcome.

A LATE START. A 2014 New York Times article by Ginia Bellafante³ described a group of community college students who were required to take a developmental math course that included formal algebra topics. According to the article, their experience was not uncommon; in the U.S. “over 60 percent of all students entering community colleges must take … developmental math courses … [that] are algebra-based and focus on linear and quadratic equations.” These students had graduated from high school, yet still struggled with the fundamentals of algebraic equations. It’s not difficult to understand why: despite its importance, only 35 percent of eighth grade students are considered proficient in algebra.⁴
By the time students reach college, it's often too late. Therefore, there are obvious benefits to starting the development of algebraic reasoning early. Since 1998, the Tufts University Early Algebra project has understood the importance of starting early. They have taught young students the time-honored topics of early mathematics in deeper, more challenging ways. Their position continues to be that children who become familiar with algebraic concepts and tools from an early age and in meaningful contexts will do better in mathematics, regardless of the evaluation criteria. Research from this project at Tufts has validated their assertion for many of the students in their program. Find details on results and impact of the Tufts University Early Algebra project here.

Only 44% of U.S. high school graduates were ready for college-level math in 2013.  

The “Why Bother?” Mindset. At a time when it is easy to access instant answers and information on the Internet using sites like Wolfram|Alpha, many students question why they need to learn algebra at all. According to mathematician David Bressoud, Wolfram|Alpha poses a new challenge to educators that extends from issues that arose when the first four-function calculators entered the mainstream. Bressoud asks, “If computers can solve … problems so efficiently, why do we drill our students in answering them? … There are important mathematical ideas behind these methods, and showing one knows how to solve these problems is one way of exhibiting working knowledge of these ideas.”

It’s therefore essential that students personally realize that algebra is a powerful tool for describing and evaluating relationships, as well as for solving meaningful problems in their world. To ensure this outcome and empower all students to reason algebraically with purpose, it’s important to create or select assessments that balance computational skill with concept application. A simple way for teachers to evaluate their assessments is to have Wolfram|Alpha “take their test.” If Wolfram|Alpha can solve enough answers correctly on the test to earn a B, then the test likely doesn’t collect enough evidence of student learning that’s aligned with all of the key outcomes in algebra.

Only 44% of U.S. high school graduates were ready for college-level math in 2013.

Source: ACT.org
TEACHER-LEVEL FACTORS. According to John Hattie’s meta-analysis in *Visible Learning*, out of the top 60 factors influencing student achievement, 57 are within the control of schools and teachers.7 Factors most within the control of teachers are the taught curriculum, classroom practices, and pedagogical decisions. It is often these aspects that can have a profound impact on the degree to which students become confident math learners who recognize that algebra is not just a collection of procedures and formulas to remember. Students need to learn that algebra is a way of thinking, reasoning, and communicating mathematically. When opportunities for Deeper Learning are integrated into lesson plans, students begin to see math in this new, more useful, and more accurate way.

The best way to engage students in algebraic thinking is to give them problems worth thinking about. Each student, classroom, and community is different, and at any given time there will be a variety of mathematical problems worth bringing into the classroom that are related to current events in politics, sports, science, or entertainment. It might also be that students will be inspired to think about hypothetical situations that have no practical application but will spark curiosity. Estimation problems are a good example: how many golf balls would fit in the gymnasium if it were filled to the ceiling? The key for teachers is to ensure that the problems are thought provoking rather than mind numbing.

The following classification framework can help teachers consider whether a math problem is likely to resonate with students.
STUDENT-LEVEL FACTORS. According to a 2015 article on seattlepi.com by Maria OCadiz, “One of the most glaring differences between good and bad math students is that the former actively participate in class, seeking to understand math vocabulary, principles and formulas, while others remain passive, hoping to memorize what they need for the test.” The article also points out that, “students who have negative thoughts, low expectations, and anxiety toward math tend to struggle in the subject because they convince themselves they can only fail at it.” Given this reality, it is important to help students reshape their opinions of themselves as math learners by helping them cultivate a growth mindset.

OCadiz’s points also reveal potential problems with the way student understanding is assessed in algebra. Just as teachers can evaluate the rigor of an assessment by letting Wolfram|Alpha take the test, they should also conduct an assessment audit to determine how well a student could perform simply by “memorizing what they need for the test.” For example, consider this typical textbook test item:

Write the slope-intercept form of a linear equation when \( m = 2 \) and \( b = 3 \).

This item can be answered correctly by simply memorizing that slope-intercept form is \( y = mx + b \). Furthermore, this item provides no more evidence of a student’s understanding of algebra than what this problem reveals about a student’s understanding of history:

Write the full name of a president when \( \text{First} = \text{George} \) and \( \text{Last} = \text{Washington} \).

As is usually the case, students value what is assessed. Thus, by collecting evidence of student learning that goes beyond merely what can be memorized, teachers can improve rigor and student expectations of what counts as algebraic thinking.
Deeper Learning for algebraic understanding

Educators and students can overcome these algebra barriers by using the principles of Deeper Learning. Deeper Learning isn’t simply a checklist; it is a set of interrelated competencies that students need in order to develop a true understanding of algebra content and processes that they can use to apply their knowledge to new and unfamiliar challenges in the classroom, in life, and at work.

Engaging students in Deeper Learning enables them to use their knowledge and skills in ways that prepare them for new challenges in school and in life.

- **STRATEGY**
  - Empower students to design their own solutions
  - Use prior knowledge to tackle unfamiliar tasks
  - Cultivate persistence as students get older
  - Honor and understand multiple points of view
  - Put each student in the driver’s seat
  - Formal algebra is formal communication
  - Critical Thinking & Problem Solving
  - Self-Directed Learning
  - Effective Communication
  - Collaboration
  - Academic Mindset

Source: Hewlett.org
The notion of Deeper Learning in mathematics is slowly being applied in classrooms across the country. Classroom teachers are focused on creating learning environments where students are engaged and thinking critically—and where they gain the confidence to persevere through challenging but achievable tasks. Supported by the Hewlett Foundation, a report found that Deeper Learning schools graduate high school students on time at rates nine percent higher than other schools. Furthermore, students at these schools are four percent more likely to enroll in four-year colleges. These outcomes are to be expected, according to The American Institutes for Research (AIR) because, “In classrooms where Deeper Learning is the focus, you find students who are motivated and challenged—who look forward to their next assignment. They apply what they have learned in one subject area to newly encountered situations in another. They can see how their classwork relates to real life.”

Deeper Learning in the digital age

Before further exploring the characteristics of Deeper Learning, it’s important to recognize the current realities of schools and classrooms. Faced with new, more rigorous standards and increasing class sizes, classroom teachers often find themselves too pressed for time to provide the Deeper Learning opportunities each student needs. According to Rose Colby, a consultant on competency education, “when learning is done on a deeper level, it takes longer to accomplish.” Like most worthy tasks, the investment of hard work results in enduring value. But effectively teaching math and algebra to a broad range of learners has become a problem of rigor and scale that can be supported by technology and blended learning models.

Teachers are strategically leveraging online learning experiences to effectively provide each student with a personalized learning experience that fosters Deeper Learning. Technologies that extend the capacity of the teacher to individualize learning and create opportunities for students to engage in meaningful situations are necessary in order to promote Deeper Learning for all students. To effectively differentiate instruction for students, technologies must have a high degree of adaptation, be based on a rigorous curriculum, and empower students to be self-directed learners. It’s also helpful if students are able to work within an environment that is engaging and enjoyable, or employs productive gamification. You can learn more about the characteristics and effectiveness of adaptive technology in the math classroom, and what criteria to look for when you are selecting digital curricula in Help all Students Excel in Math with Adaptive Learning Technology.
Six ways to use Deeper Learning* and technology to support both math educators and students

1. CRITICAL THINKING AND PROBLEM SOLVING

Students think critically, analytically, and creatively. They know how to find, evaluate, and synthesize information to construct arguments. They can design their own solutions to complex problems.

STRATEGY: Empower Students to Design Their Own Solutions. All algebraic reasoning, reflection, communication, and self-direction begin with a student’s independent, critical thinking. Because each student alone is the only person who can truly make sense of mathematics ideas, students need opportunities to think analytically and creatively. Too often in mathematics, students are deprived of the opportunity to genuinely create their own solutions to meaningful problems.

Consider for a moment how you would find the sum of 27 + 52. Here are possible ways a student might think about that problem:

- 20 and 50 makes 70. And 7 and 2 makes 9. So it’s 79.
- 7 and 2 makes 9. 20 and 50 makes 70. So it’s 79.
- 25 and 50 is like 3 quarters—so that’s 75. Then there’s 4 more. So it’s 79.
- 27 and 50 is 77. Then add 2 more. So it’s 79.
- 27 and 2 is 29. Then add 50. So it’s 79.
- 50 and 30 is 80. But that’s counting 1 too many. So it’s 79.

To make sense of numbers and addition, students need the ability to think through their own solutions to this problem and represent them with an interactive number line and other manipulatives. Truly interactive virtual manipulatives with adaptive technology enable students to genuinely show what they’re thinking. The right virtual manipulatives combined with adaptive technology can provide students with a safe, fun environment to take intellectual risks and truly figure problems out for themselves at their level of achievable challenge.

* All Deeper Learning competency definitions sourced from Hewlett.org.
2. SELF-DIRECTED LEARNING

Students set goals, monitor their own progress, and reflect on their own strengths and areas for improvement. They learn to see setbacks as opportunities for feedback and growth. Students who learn through self-direction are more adaptive than their peers.

STRATEGY: Put Each Student in the Driver’s Seat. Often, teachers are too constrained by schedules and pacing calendars to empower students to self-direct a portion of their learning. A pacing calendar is neither self-directed for students nor adaptive to each learner’s needs. And because the typical classroom advances linearly, there aren’t many opportunities for students to explore math topics of interest, advance to new topics, or spend more time working with topics that they found challenging in the past. Even though here is no single “just right” lesson at any given moment for a student, there are always multiple “just right” lessons that students should be able to choose from.

Leveraging technology as one element of more personalized Deeper Learning enables students to more effectively overcome barriers to their success because they can have a safe, independent learning experience as they learn from mistakes and choose appropriate lessons. With adaptive technologies that engage students in conceptual thinking—with continuous formative assessment and scaffolding—there are new opportunities to empower students with some element of control over the pace and path of their learning, while benefitting from “just in time” feedback. For example, a Grade 3 student could be working on Grade 2 Place Value topics, Grade 3 Fractions topics, and Grade 4 Multiplication lessons because these topics aren’t directly mathematical pre-requisites for each other. When students are appropriately challenged and have some element of choice, they develop habits and mindsets that help their overall mathematical thinking. To learn more, watch Seamless Formative Assessment for Personalized Learning.

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3. EFFECTIVE COMMUNICATION

Students communicate effectively in writing and in oral presentations. They structure information in meaningful ways, listen to and give feedback, and construct messages for particular audiences.

STRATEGY: Formal Algebra is Formal Communication. If you ask 10 adults to describe their memories of Algebra class from when they were in school, you’ll probably find that most of them say, “There were so many rules to remember.” And it’s likely that none of them will say something like, “I remember learning that algebra is a way to communicate about equivalence using symbols instead of words.” This big idea is critical, and is a reason why every student using the variable \( x \) should be able to explain that the variable represents something unknown. It’s important to note that a variable is not necessarily representing a missing number, because when students begin composing functions such as \( f(g(x)) \) when \( f(x) = 2x \) and \( g(x) = 3x + 2 \), they need to understand that \( (3x + 2) \) can be substituted for \( x \).

When students realize that two big ideas in algebra are “equivalence” and “communicating equivalence,” they will be able to make better sense of the “rules”—which are often conventions of communication that mathematicians have agreed to. Author Andrea diSessa, in his book Changing Minds, provides a useful example that highlights not only the importance of symbolic algebra, but also of how teenagers in 2015 take for granted that they are standing on the shoulders of giants. diSessa shares several theorems developed by Galileo Galilei (1564–1642). Here is one of them:

*If a moving particle, carried uniformly at constant speed, traverses two distances, then the time intervals required are to each other in the ratio of these distances (page 13).*

Here, Galileo is describing a basic algebraic relationship: \( d = rt \). In Galileo’s time, symbolic algebra and the equal sign hadn’t been invented yet, so his ability to communicate algebraic relationships was understandably cumbersome. This theorem simply states that when \( r_1 = r_2 \) then \( d_1 / d_2 = t_1 / t_2 \).

With new digital technologies, students are able to not only communicate with and manipulate algebraic symbols in ways that were never before possible, but they are also able to connect those symbols to visualizations and virtual manipulatives. Using innovative representations including virtual number lines and open arrays, students can communicate their solution strategies visually and with equations in powerful ways.
4. COLLABORATION

Collaborative students work well in teams. They communicate and understand multiple points of view, and they know how to cooperate to achieve a shared goal.

**STRATEGY: Honor and Understand Multiple Points of View.** Just as it’s important for students to design their own solutions and communicate their thinking, students also need to collaborate well. Fortunately, in classrooms where students are empowered to design and communicate, they more easily understand that there are multiple points of view and ways to approach a particular problem. Furthermore, they recognize the value of contributions made by their peers, which broadens their perspective.

Thriving communities in math classrooms need tools that enable them to achieve the shared goal of success and growth for every student. The same interactive virtual manipulatives that can provide a personalized experience can also be leveraged by teachers to engage small groups or the whole class in meaningful dialogue. By introducing a thought-provoking problem and inviting students to use the manipulative to represent their own solution strategies and critique those of others, teachers can create powerful dialogue that deepens everyone’s understanding.

By fostering algebraic exploration at an early age and leveraging digital tools that support both concrete and symbolic mathematical representations, teachers can gain valuable insights about how students are thinking and developing both their confidence and their communication. Some additional practical advice for use in the classroom is in the webinar *Concrete to Abstract: Preparing Students for Formal Algebra.*
5. ACADEMIC MINDSET

Students with an academic mindset have a strong belief in themselves. They trust their own abilities and believe their hard work will pay off, so they persist to overcome obstacles. They also learn from and support each other. They see the relevance of their schoolwork to the real world and their own future success.

STRATEGY: Cultivate Persistence as Students Get Older. In general, pre-school and kindergarten students have an academic or growth mindset. This mindset is natural, but as students age and become more self-aware and conscious of the opinions of others, it begins to diminish. Therefore, teachers in upper elementary and middle grades must make a concerted effort to cultivate or re-instill this mindset with students.

Teachers nurture this mindset in classrooms every day, but students also need experiences that cultivate their mindset independently. With more personalized classrooms and learning technology, students can develop both their math understanding and their ability to persist. For older students in mathematics—and in algebra especially—it’s essential that they realize that while the content might be challenging, it is accessible to them and they can make sense of it with the right learning experiences. Teachers should continually remind students that their struggle or confusion is essential and valuable for learning. As Rhett Allain, an Associate Professor of Physics at Southeastern Louisiana University and author for Wired rightly notes, “Confusion is the sweat of learning.” When students realize that thinking hard in algebra is like working out hard for sports or fitness, they are better positioned to develop a growth mindset.

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6. MASTERING CORE ACADEMIC CONTENT

Students understand key principles and procedures, recall facts, use the correct language, and draw on their knowledge to complete new tasks.

STRATEGY: Content Proficiency is the End, Not the Means. When students engage algebra concepts in ways that align with the first five elements of Deeper Learning, content expertise is far more likely to be the resulting outcome. Too often in mathematics, skills and content are front-loaded because there’s an assumption that students couldn’t possibly be ready to think critically until they have “mastered basic skills.” Consider what Wiggins and McTighe say about this flawed approach—which they refer to as the “climb the ladder” model—and its negative impact on both learning and mindset:

One practical problem with the “climb the ladder” view directly affects lower-achieving students. Because they are less likely to have acquired the basics on the same schedule as more advanced learners, struggling learners are often confined to an educational regimen of low-level activities, rote memorization of discrete facts, and mind-numbing skill-drill worksheets. The unfortunate reality is that many of these students will never get beyond the first rung of the ladder and, therefore, have minimal opportunities to actually use what they are learning in a meaningful fashion (page 45).

Content is not the prerequisite for critical thinking; rather it is critically thinking about content that results in content “mastery.”

The best educational technologies can enable students to extend their learning by focusing on the underlying principles and foundational “big ideas” in algebra rather than memorization. When these digital tools incorporate the principles of Deeper Learning, classroom teachers can trust that students using technology are designing their own solutions as they develop content expertise. Adaptive technology can also provide each student with learning experiences that are more closely aligned with his or her individual needs. And if students can access technology both in school and at home, they have multiple opportunities over time to apply knowledge in a range of “just right” challenging tasks. Watch Closing Achievement Gaps in Algebra I and Algebra Readiness: Equipping K–8 Students for Success for some ideas around providing personalized support for learners through technology.
“Algebra-ready” students don’t just memorize formulas—they are mathematical communicators who understand relationships and know enough about equivalence to derive formulas from memory. They are mathematical thinkers who are eager and able to design their own solutions and drive their own learning. Effectively preparing students for success in algebra means ensuring they develop independent critical-thinking skills and the confidence in their ability to reason logically, communicate algebraically, and persist through mathematical challenges.

DreamBox Learning Math is an adaptive technology that helps teachers and schools engage students in algebraic thinking and Deeper Learning as early as kindergarten, and throughout their elementary and early middle school years. When using DreamBox, students model relationships and connect with mathematical ideas through digital manipulatives and lessons that go beyond what can be accomplished with pencil and paper. By using these highly engaging conceptual tools, students can make sense of the most important ideas in algebra by designing their own solutions and justifying their thinking. They become truly fluent mathematicians. Unlike other math software programs and digitized textbooks that present math as a sequence of linear skills to remember and rules to follow, DreamBox’s transformative digital experiences engage students in algebraic reasoning that results in conceptual understanding, procedural fluency, and the capacity for strategic and creative problem solving.

Students can successfully learn algebraic concepts more deeply than ever before. We can reverse the troubling trends of algebra proficiency and help all students become proficient in algebra and achieve their goals in school and in life.
REFERENCES

9. Ibid.

DreamBox Learning, Inc. was founded in Bellevue, Washington, and launched its first online learning product in January 2009. DreamBox Learning® Math has won more than 35 top education and technology industry awards and is in use in all 50 U.S. states and throughout Canada. The DreamBox® platform offers a groundbreaking combination of Intelligent Adaptive Learning™ technology, a rigorous K–8 mathematics curriculum, and a highly motivating learning environment. DreamBox in English and Spanish captures every decision a student makes while working in the program and adjusts the student’s learning path appropriately, providing millions of individualized learning paths, each one tailored to the student’s unique needs. DreamBox supports teachers and their practice in every type of learning environment. For more information about DreamBox Learning Math and the DreamBox Math for iPad® app, please visit DreamBox.com.

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